Incentivizing Effort in Interdependent Security Games Using Resource Pooling*

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ABSTRACT

We consider an InterDependent Security (IDS) game with networked agents and positive externality where each agent chooses an effort/investment level for securing itself. The agents are interdependent in that the state of security of one agent depends not only on its own investment but also on the other agents' effort/investment. Due to the positive externality, the agents under-invest in security which leads to an inefficient Nash equilibrium (NE). While much has been analyzed in the literature on the under-investment issue, in this study we take a different angle. Specifically, we consider the possibility of allowing agents to pool their resources, i.e., allowing agents to have the ability to both invest in themselves as well as in other agents. We show that the interaction of strategic and selfish agents under resource pooling (RP) improves the agents' effort/investment level as well as their utility as compared to a scenario without resource pooling. We show that the social welfare (total utility) at the NE of the game with resource pooling is higher than the maximum social welfare attainable in a game without resource pooling but by using an optimal incentive mechanism. Furthermore, we show that while voluntary participation in this latter scenario is not generally true, it is guaranteed under resource pooling.

CCS CONCEPTS

• **Networks → Network economics**; • **Security and privacy → Network security**.

1 INTRODUCTION

The increasing rate and scale of cyber crime is placing significant pressure on organizations to improve their security posture. At the same time, the interdependent nature of cyber risks means one's state of security is not just the result of one's own security practices and investments, but of others' connected to it, e.g., through attack propagation and supply chain relationships. Decision making in such a scenario has often been modeled as an InterDependent Security (IDS) game [18]. The most critical issue that arises in IDS games is free-riding where an entity under-invests in security and takes advantage of others' efforts. As a result, the Nash equilibrium (NE) in IDS games is inefficient and individuals' investment in security is below the optimum [14, 27].

To address the free-riding issue and incentivize individuals to improve their security investment, various mechanisms have been proposed. [7] shows that bonus and penalty based on agents' security outcome can improve network security. [13] and [15] show that cyber insurance in the presence of a quantitative security assessment (pre-screening) is able to improve the security investment and address the free-riding issue. Ioannidis *et al.* in [11] show that public coordination under the guidance of a well-informed steward can improve the resilience of the system to attacks. In [20], the Pivotal (VCG) and Externality mechanisms are analyzed (both are in the form of a taxation/subsidy mechanism) to induce socially optimal outcome in IDS games; however, it also shows that no tax mechanism can simultaneously satisfy both budget balance and voluntary participation constraints. This is because security is a *non-excludable* public good and individuals continue to benefit from other's effort even if they unilaterally opt out of the mechanism.

All of the above studies assume the existence of a central entity, a social planner, a coordinator, or a steward. In this study, different from existing literature, we shall take a different approach to inducing socially desirable or optimal outcomes in this type of IDS games. Specifically, we consider the absence of such a central entity, and instead model the presence of resource pooling (RP) by allowing agents to have the ability to both invest in themselves as well as in other agents, so that they can choose to not only improve its own but also others' state of security. This modeling choice leads to a different IDS game, referred to as the RP-augmented IDS game, or simply RP-IDS below. In practice, exerting efforts on other agents' behalf has context dependent interpretations, such as providing product/service discounts to customers by a service provider, as well as funding open source development. Note that both IDS game and RP-IDS are non-cooperative games where agents selfishly choose their actions to maximize their own utility. Thus our model is different from that of a cooperative game [24–26] where players form coalitions and choose actions to maximize the utility of the coalition they belong to. A cooperative game can lead to improved network security as compared to a non-cooperative one if the cost of forming coalitions is sufficiently low, but forming coalition is not always possible due to cultural, economical, or social reasons [22].

Specifically, we study the IDS game with a weighted total effort and quadratic cost model under two scenarios: (i) no RP (the original IDS game), where each agent exerts effort only to improve his own security; and (ii) with RP (RP-IDS), where selfish agents pool their resources. Our main findings are summarized as follows.

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1) Both games have a unique NE. At the NE of the RP-IDS game, every agent obtains higher utility as compared to that under the NE of the IDS game.

2) The social welfare (measured by total utility) at the NE of the RP-IDS game is higher than that under the *socially optimal* outcome of the IDS game, induced by mechanisms such as VCG and externality mechanisms [20]. In other words, as a mechanism, RP outperforms these tax-based mechanisms.

3) While the VCG and externality mechanisms cannot guarantee voluntary participation while imposing budget balance [20], we show that in the RP-IDS game no agent will unilaterally opt out of resource pooling (while continue to be part of the IDS game), thereby ensuring voluntary participation.

Related Works

Distributed mechanism framework has been proposed to induce socially optimal outcome in a distributed manner, i.e., message transmission is performed locally, and mechanism/tax functions depend on messages from neighboring agents [8, 9, 23]. Even though distributed mechanisms are viable options to implement the socially optimal outcome without a central planner, they still cannot be used in IDS games because they are in the form of taxation mechanism and not able to satisfy the notion of voluntary participation [20].

Outside the incentive context, IDS games have been extensively studied in the literature [2, 10, 12, 17, 19]; we reference some of the more relevant ones below. Ann Miura-Ko *et al.* [19] consider a linear influence network and find a condition on the dependence matrix to guarantee the existence and uniqueness of the NE. Hota and Sundaram in [10] consider IDS games under behavioral probability weighting and show that security risk can be reduced by such weighting strategies. [2] shows that the under-investment issue similarly exists in a two-stage game model. [17] examines the relationship between risk exposure and agents' degrees in the dependence graph.

The most related work to the present paper is [16] which studies the role of resource pooling in public good provision games with limited resources. The main conclusion of [16] is that resource pooling under limited resources is not able to induce socially optimal outcomes, for which an incentive mechanism is needed.

In the remainder of the paper, we present the IDS game model without RP, and the RP-IDS game model, and their associated analysis, in Sec. 2 and 3, respectively. A number of discussions are given in Sec. 4. Sec. 5 concludes the paper.

2 INTERDEPENDENT SECURITY GAME WITHOUT RESOURCE POOLING (IDS)

Consider n agents on a directed, weighted graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, X)$, where $\mathcal{V} = \{1, 2, \cdots, n\}$ is the set of n agents, $\mathcal{E} \subseteq \{(i,j)|i \neq j,\ i,j \in \mathcal{V}\}$ the set of edges between them, and $X = [x_{ij}]_{n\times n}$ the adjacency weight matrix of this graph, where $x_{ij} > 0, (i,j) \in \mathcal{E}$ is the edge weight, $x_{ij} = 0, (i,j) \notin \mathcal{E}$, and $x_{ii} = 0, i \in \mathcal{V}$. An edge $(i,j) \in \mathcal{E}$ indicates that agent i *depends on* agent j (or agent j *influences* i) with the degree of dependence given by edge weight x_{ij}. Dependence need not be symmetrical, i.e., $x_{ij} \neq x_{ji}$ in general. We assume $x_{ii} = 0, i \in \mathcal{V}$. Agent i exerts effort $e_i \geq 0$ towards securing himself, incurring cost $b_i \cdot e_i^2$ ($b_i > 0$ a constant). Given effort profile $\boldsymbol{e} = [e_1, e_2, \cdots, e_n]^T$, agent i has utility

$$u_i(e_i, e_{-i}) = -l_i + a_i \cdot e_i + e_i \cdot \left(\sum_{j=1}^{n} x_{ij} e_j\right) - b_i \cdot e_i^2, \tag{1}$$

where e_{-i} denotes all elements in $\boldsymbol{e}$ excluding e_i, $-l_i$ a nominal loss agent i suffers without any effort, $a_i \cdot e_i$, $a_i \geq 0$, the benefit it derives from effort e_i, and $e_i \cdot x_{ij} \cdot e_j$ the benefit it derives from other agents' efforts. This term suggests that an agent who does not exert effort also does not benefit from other's efforts. This may be interpreted as implying that the type of security products or technologies agents use are complementary. Note that $x_{ij} \geq 0$ indicates a case of positive externality between agents i and j; see e.g., [6] for IDS games with negative externalities. This is a form of the quadratic utility function widely used in the literature of network games [3, 5] and IDS games [4, 21]; it can be viewed as a second-order approximation of any utility function.

The interaction of agents induces a game, denoted as $G = \{\mathcal{V}, \{u_i(.)\}_{i\in\mathcal{V}}, A = [0, +\infty)^n\}$, where A is the action space. In the rest of the paper, we shall use the terms *exerted effort*, *actions* and *security investments* interchangeably. For convenience of notation, when comparing two games given by the same $\mathcal{V}, \mathcal{E}$ but different weight matrices X_1 and X_2, we will denote the resulting games as $G(X_1)$ and $G(X_2)$, respectively. Next we analyze the equilibrium of game G.

2.1 Equilibrium Analysis

Let $Br_i(e_{-i})$ denote the best response function of agent i. Using the first order condition we have

$$\begin{aligned} Br_i(e_{-i}) &= \arg\max_{e\geq 0}\ u_i(e, e_{-i}) \\ &= \frac{a_i}{2b_i} + \frac{1}{2b_i}\sum_{j=1}^{n} x_{ij} e_j\ . \end{aligned} \tag{2}$$

We will primarily focus on pure strategy Nash equilibrium (NE), and for simplicity of exposition for the rest of the paper Nash equilibrium refers to a pure strategy NE. An NE is the fixed point of the best response mapping. Let $\hat{\boldsymbol{e}}$ denote the agents' effort at the NE of game G; then $\hat{\boldsymbol{e}}$ satisfies the following equations:

$$\begin{aligned} 2b_i\hat{e}_i - \sum_{j=1}^{n} x_{ij}\hat{e}_j &= a_i,\ \ i = 1, 2\cdots, n \\ \text{or}\quad (2\cdot B - X)\cdot \hat{\boldsymbol{e}} &= \boldsymbol{a}, \end{aligned} \tag{3}$$

where B is a matrix with b_i's on its main diagonal and zeros everywhere else, and $\boldsymbol{a} = [a_1, a_2, \cdots, a_n]^T$.

We make the following assumption on cost b_i to ensure that the effort levels are bounded at the NE. More discussion on this assumption is provided in Section 4.1.

Assumption 1. $2b_i \geq \sum_{j=1}^{n} x_{ij},\ \forall i \in \mathcal{V}$.

Under this assumption, we have the following lemma on the best response mapping and the NE of game G.

Theorem 2.1. *Under Assumption 1, matrix $(2B - X)$ is invertible and $\hat{\boldsymbol{e}} = (2\cdot B - X)^{-1}\cdot \boldsymbol{a}$ is the unique NE of game G.*

Proof. See Appendix. ■

Note that Theorem 2.1 holds for any non-negative vector $\boldsymbol{a}$, which leads to the following corollary.

Corollary 2.2. *Under Assumption 1, all entries of matrix $(2 \cdot B - X)^{-1}$ are non-negative. Furthermore, let X and $\tilde{X}$ be two adjacency matrices over the same $\mathcal{V}$ and $\mathcal{E}$. Consider the games $G(X)$ and $G(X + \tilde{X})$, and their respective NE $\hat{\boldsymbol{e}}$ and $\tilde{\boldsymbol{e}}$. If $2b_i \geq \sum_{j=1}^{n} \left[x_{ij} + \tilde{x}_{ij}\right]$, then $\tilde{\boldsymbol{e}} \geq \hat{\boldsymbol{e}}$.*[1] *In other words, agents exert higher effort at the NE given stronger externality.*

Proof. Let $\mathbf{0} \in \mathbb{R}^n$ be a zero vector. By Theorem 2.1, we know that $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \geq \mathbf{0}$ for any non-negative vector $\tilde{\boldsymbol{a}}$. Set $\tilde{a}_i = 1$ and $\tilde{a}_j = 0,\ \forall j \neq i$ and $\tilde{\boldsymbol{a}} = [\tilde{a}_1, \cdots, \tilde{a}_n]^T$. Then, $(2 \cdot B - X)^{-1} \cdot \tilde{\boldsymbol{a}} \geq \mathbf{0}$ is the ith column of $(2 \cdot B - X)^{-1}$. Because i is arbitrary, all columns of $(2 \cdot B - X)^{-1}$ are non-negative. Moreover, we have,

$$\begin{aligned} (2B - X) \cdot \hat{\boldsymbol{e}} &= \boldsymbol{a} \\ (2B - X - \tilde{X}) \cdot \tilde{\boldsymbol{e}} &= \boldsymbol{a} \implies \\ \tilde{\boldsymbol{e}} = (2B - X)^{-1} \cdot \boldsymbol{a} &+ (2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}} \\ &= \hat{\boldsymbol{e}} + \underbrace{(2B - X)^{-1} \cdot \tilde{X} \cdot \tilde{\boldsymbol{e}}}_{\geq \mathbf{0}} \geq \hat{\boldsymbol{e}} \end{aligned} \quad (4)$$

■

2.2 Socially Optimal Outcome

We now consider the socially optimal effort levels for the IDS game. Denote by $\boldsymbol{e}^* = [e_1^*, e_2^*, \cdots, e_n^*]$, the socially optimal effort profile maximizes the total utility:

$$\boldsymbol{e}^* \in \arg\max_{\boldsymbol{e} \in A} \sum_{i=1}^{n} u_i(e_i, e_{-i}) \ . \quad (5)$$

To ensure the existence of a socially optimal strategy, we make the following assumption (see Section 4.1 for more discussion).

Assumption 2. $2b_i \geq \sum_{j=1}^{n} \left[x_{ij} + x_{ji}\right],\ \forall i \in \mathcal{V}$.

Theorem 2.3. *Let $\hat{\boldsymbol{e}}$ be the effort level at the NE of game G and $\boldsymbol{e}^*$ be the socially optimal effort level. Under Assumption 2 we have:*

(1) $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$;
(2) $e_i^* \geq \hat{e}_i,\ \forall i$.

That is, every agent exerts higher effort at the socially optimal solution compared to the NE.

Proof. See Appendix. ■

Remark: The above shows that the socially optimal effort profile of game $G(X)$, given by $\boldsymbol{e}^* = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$, also happens to be the NE of game $G(X + X^T)$. Also note that for game $G(X)$, while the total utility under $\boldsymbol{e}^*$ is higher than that under the NE $\hat{\boldsymbol{e}}$, this may or may not be true for agents' individual utility, as the following example shows.

Example 2.4. Consider the following IDS game:

$$\begin{aligned} n &= 2,\ b_1 = b_2 = 1,\ a_1 = a_2 = 1 \\ x_{12} &= 0.1,\ x_{21} = 0.9,\ l_1 = l_2 = 1 \\ \hat{\boldsymbol{e}} &= (2B - X)^{-1} \cdot \boldsymbol{a} = [0.5371\ 0.7417]^T \\ u_1(\hat{\boldsymbol{e}}) &= -0.7115,\ u_2(\hat{\boldsymbol{e}}) = -0.4499 \\ \boldsymbol{e}^* &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = [1\ 1]^T \\ u_1(\boldsymbol{e}^*) &= -0.9000,\ u_2(\boldsymbol{e}^*) = -0.1000 \end{aligned} \quad (6)$$

In this example, agent 1 has higher influence on agent 2 ($x_{21} > x_{12}$); agent 2 benefits from socially optimal effort ($u_2(\boldsymbol{e}^*) > u_2(\hat{\boldsymbol{e}})$), while agent 1's utility worsens even though it exerts higher effort under $\boldsymbol{e}^*$.

Example 2.5. Consider the following IDS game where both agents benefit from the socially optimal outcome:

$$\begin{aligned} n &= 2,\ b_1 = b_2 = 1,\ a_1 = a_2 = 1 \\ x_{12} &= x_{21} = 0.5,\ l_1 = l_2 = 1 \\ \hat{\boldsymbol{e}} &= (2B - X)^{-1} \cdot \boldsymbol{a} = [\frac{2}{3}, \frac{2}{3}]^T \\ u_1(\hat{\boldsymbol{e}}) &= u_2(\hat{\boldsymbol{e}}) = -\frac{5}{9} = -0.5555 \\ \boldsymbol{e}^* &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = [1\ 1]^T \\ u_1(\boldsymbol{e}^*) &= u_2(\boldsymbol{e}^*) = -0.5 \end{aligned} \quad (7)$$

These examples show that socially optimal outcome is not necessarily desirable to all agents. Mechanism design in the context of IDS games aims to incentivize agents to exert higher effort than that under the NE. In the next section, we will examine the impact of introducing resource pooling as a mechanism to improve agents' effort and social welfare.

3 INTERDEPENDENT SECURITY GAME WITH RESOURCE POOLING (RP-IDS)

Consider the same IDS game setting. Let $\boldsymbol{e}_i = [e_{i1}, e_{i2}, \cdots, e_{in}]^T$ be the action of agent i where $e_{ij} \geq 0$ denotes the effort exerted by agent i on behalf of agent j. Moreover, agent i incurs cost $b_j \cdot e_{ij}^2$ by exerting effort e_{ij} on behalf of agent j, i.e., the cost of exerting an effort on behalf of agent j depends on j.[2] Let $E = [\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_n]^T$ be an $n \times n$ matrix that denotes the effort profile, and let $E_i = \sum_{j=1}^{n} e_{ji}$ denote the total effort exerted on behalf of agent i. Agent i's utility given profile E is:

$$\begin{aligned} v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) &= -l_i + a_i(\sum_{j=1}^{n} e_{ji}) - \sum_{k=1}^{n} b_k \cdot e_{ik}^2 \\ &+ (\sum_{j=1}^{n} e_{ji}) \cdot \sum_{k=1}^{n} x_{ik} \cdot (\sum_{r=1}^{n} e_{rk}) \\ &= -l_i + a_i E_i + E_i \cdot \sum_{j=1}^{n} x_{ij} E_j - \sum_{k=1}^{n} b_k \cdot e_{ik}^2 . \end{aligned}$$

The interaction of agents induces the RP-IDS game $G_{rp} = \left\{\mathcal{V}, \{v_i\}_{i \in \mathcal{V}}, A_{rp} = [0, +\infty)^{n^2}\right\}$, where A_{rp} is the action

[1] $\boldsymbol{v} = [v_1 \cdots v_n]^T \geq \boldsymbol{\theta} = [\theta_1 \cdots \theta_n]^T$ means that $v_i \geq \theta_i,\ \forall i$.

[2] An example of this is one firm providing security training for employees of another firm; the incurred training cost depends on the trainees' education level.

space under resource pooling. By first order condition the best response function of agent i satisfies the following:

$$\begin{aligned} \boldsymbol{e}_i &= Br_i(\boldsymbol{e}_{-i}) \\ e_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot E_k}{2b_i} \\ e_{ij} &= \frac{x_{ij} \cdot E_i}{2b_j}, \ \forall j \neq i \end{aligned} \tag{8}$$

Let $\hat{E} = [\hat{e}_{ij}]_{n \times n}$ be the NE of game G_{rp} and $\hat{E}_i = \sum_{j=1}^n \hat{e}_{ji}$ the total effort exerted on behalf of agent i at the NE. We have the following lemma on effort profile $\hat{E}$.

Lemma 3.1. *Assume that game G_{rp} has at least one Nash equilibrium. The effort profile $\hat{E}$ at the NE satisfies the following system of equations,*

$$(2B - X - X^T) \cdot \begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} = \boldsymbol{a} \ .$$

Proof. As effort profile $\hat{E}$ is the fixed point of the best response mapping, we have,

$$\begin{aligned} \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \hat{E}_k}{2b_i} \\ \hat{e}_{ji} &= \frac{x_{ji} \cdot \hat{E}_j}{2b_i} \ \forall j \neq i \implies \end{aligned}$$

by adding above equations:

$$\begin{aligned} 2b_i \cdot \hat{E}_i &= a_i + \sum_{j=1}^n (x_{ij} + x_{ji}) \hat{E}_j \ \ \forall i \in \mathcal{V} \\ \implies \boldsymbol{a} &= (2B - X - X^T) \cdot \begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} \end{aligned} \tag{9}$$

■

Theorem 3.2. *Under Assumption 2, $(2B - X - X^T)$ is invertible and game G_{rp} has a unique NE given as follows:*

$$\begin{aligned} \begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} &= (2B - X - X^T)^{-1} \cdot \boldsymbol{a} \\ \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \hat{E}_k}{2b_i} \\ \hat{e}_{ij} &= \frac{x_{ij} \cdot \hat{E}_i}{2b_j}, \ \forall j \neq i \end{aligned} \tag{10}$$

Proof. Similar to the proof of Theorem 2.1, we can show that if $2b_i \geq \sum_{j=1}^n x_{ij} + x_{ji}, \forall i$, then all eigenvalues of matrix $(2B - X - X^T)$ are non-zero. Therefore, matrix $(2B - X - X^T)$ is invertible. By Corollary 2.2, all entries of $(2B - X - X^T)^{-1}$ are non-negative and $\left[\hat{E}_1 \cdots \hat{E}_n\right]^T = (2B - X - X^T)^{-1} \cdot \boldsymbol{a}$ is a non-negative vector. Moreover, by the best response mapping provided in (8), we know that $\hat{e}_{ij}$ can be calculated by the following,

$$\begin{aligned} \hat{e}_{ii} &= \frac{a_i}{2b_i} + \frac{\sum_{k=1}^n x_{ik} \cdot \hat{E}_k}{2b_i} \geq 0 \\ \hat{e}_{ij} &= \frac{x_{ij} \cdot \hat{E}_i}{2b_j} \geq 0, \ \forall j \neq i \end{aligned} \tag{11}$$

Therefore, the fixed point of the best response mapping is non-negative and unique, implying the NE of game G_{rp} is unique and can be found by (10). ■

Remark: It is worth pointing out that for the same weight matrix X, the *total* effort exerted by each agent, $[\hat{E}_1, \ \hat{E}_2, \cdots, \hat{E}_n]$, at the NE of the RP-IDS game G_{rp} is the same as the socially optimal effort of the IDS game G. That is,

$$\begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} = (2B - X - X^T)^{-1} \cdot \boldsymbol{a} = \boldsymbol{e}^* \underbrace{\geq}_{\text{By Theorem 2.3}} \hat{\boldsymbol{e}} \ . \tag{12}$$

In other words, the introduction of resource pooling incentivizes agents to boost their effort to the socially optimal levels for game G. Note that the game G_{rp} has its own socially optimal solution as we discuss in Section 4.3.

Next we show that every agent at the NE of game G_{rp} obtains a higher utility than that attained at the NE of game G, i.e., resource pooling improves the utility for all agents.

Theorem 3.3. *Let $\hat{E} = [\hat{e}_{ij}]_{n \times n}$ be the NE of G_{rp} and $\hat{\boldsymbol{e}}$ be the effort profile at the NE of game G. Under Assumption 2, We have:*

$$v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}}), \ \ \forall i \in \mathcal{V} \ . \tag{13}$$

Proof. Let $\tilde{\boldsymbol{e}}_i$ be a vector with length n and all its elements are zero except entry i which is equal to $\hat{e}_i$ (effort level of agent i at NE of game G). By definition of Nash equilibrium we have,

$$v_i(\hat{E}) \geq v_i(\tilde{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}). \tag{14}$$

As $\hat{E}_i \geq \hat{e}_i, \ \forall i$, by (10) and (3) we have $\hat{e}_{ii} \geq \hat{e}_i$. Moreover,

$$\begin{aligned} v_i(\tilde{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}) = -l_i + a_i \cdot \hat{e}_i + a_i \sum_{k \neq i}^n \hat{e}_{ki} - b_i \cdot (\hat{e}_i)^2 \\ +(\hat{e}_i + \sum_{k \neq i}^n \hat{e}_{ki}) \cdot \sum_{j=1}^n \left(x_{ij} \cdot (\sum_{k \neq i} \hat{e}_{kj}) \right) \geq \\ -l_i + a_i \cdot \hat{e}_i - b_i \cdot (\hat{e}_i)^2 + \hat{e}_i \cdot \sum_{j=1}^n x_{ij} \cdot \hat{e}_j = u_i(\hat{e}_i, \hat{e}_{-i}) \end{aligned} \tag{15}$$

By (14) and (15), $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}}) \ \forall i \in V$. ■

The following theorem shows that social welfare at the NE of game G_{rp} is higher than the maximum social welfare of game G, even though the total effort exerted by each agent is the same under both as noted earlier.

Theorem 3.4. *Let $\hat{E}$ be the effort profile at the NE of game G_{rp} and $\boldsymbol{e}^*$ be the socially optimal effort profile in game G. Under Assumption 2 we have,*

$$\sum_{i=1}^n v_i(\hat{E}) \geq \sum_{i=1}^n u_i(\boldsymbol{e}^*) \ .$$

Proof.

$$\sum_{i=1}^{n} v_i(\hat{E}) = \sum_{i=1}^{n}\left(-l_i + a_i\hat{E}_i - b_i \cdot \left[\sum_{j=1}^{n}\hat{e}_{ji}^2\right] + \hat{E}_i \cdot \left[\sum_{j=1}^{n} x_{ij}\cdot \hat{E}_j\right]\right)$$

By (12), $(e_i^*)^2 = \hat{E}_i^2 = (\sum_{j=1}^{n}\hat{e}_{ji})^2 \geq \sum_{j=1}^{n}(\hat{e}_{ji})^2$, and $\hat{E}_i = e_i^*$. Therefore,

$$\begin{aligned}\sum_{i=1}^{n} v_i(\hat{E}) &\geq \sum_{i=1}^{n}\left(-l_i + a_i\hat{E}_i - b_i\cdot\hat{E}_i^2 + \hat{E}_i\cdot\left[\sum_{j=1}^{n}x_{ij}\cdot\hat{E}_j\right]\right)\\ &= \sum_{i=1}^{n} u_i(\boldsymbol{e}^*). \end{aligned} \tag{16}$$

■

We conclude this section by highlighting the role of resource pooling in the IDS game.

1) At the NE, with resource pooling (game G_{rp}) agents exert higher effort (for themselves and for others) and experience higher utility than the case without resource pooling (game G); i.e., $\hat{E}_i \geq \hat{e}_i$, and $v_i(\hat{E}) \geq u_i(\hat{\boldsymbol{e}})$.

2) Resource pooling induces agents to exert socially optimal levels of effort (under game G), while improving the social welfare as it allows more judicious spreading of efforts; e.g., $\hat{E} = \boldsymbol{e}^*$ and $\sum_{i=1}^{n} v_i(\hat{E}) \geq \sum_{i=1}^{n} u_i(\boldsymbol{e}^*)$.

4 DISCUSSION

4.1 On Assumption $2b_i > \sum_{j=1}^{n} x_{ij}$

Throughout the analysis we have used the following assumptions:

- Existence and uniqueness of NE for game G: $2b_i > \sum_{j=1}^{n} x_{ij}, \forall i$
- Existence and uniqueness of socially optimal strategy profile in game G: $2b_i > \sum_{j=1}^{n} x_{ij} + x_{ji}, \forall i$
- Existence and uniqueness of NE profile in game G_{rp}: $2b_i > \sum_{j=1}^{n} x_{ij} + x_{ji}, \forall i$

The reason behind these assumptions is to prevent the model from becoming pathological: if the cost of effort is sufficiently low, then there may not exist NE or socially optimal strategy, and it may be beneficial for the agents to exert very high effort with unbounded utility.

Example 4.1. Consider a network with $x_{ii} = 0,\ x_{ij} = \frac{1}{n-1}\ \forall i, j \in V,\ i \neq j$ and $b_i = 1$. Under these parameters Assumption 2 does not hold. Moreover, set $e_i = r,\ \forall i \in V$. We have:

$$\begin{aligned}\sum_{i=1}^{n} u_i(\boldsymbol{e}) &= \sum_{i=1}^{n}\left(-l_i + (r)a_i - b_i\cdot r^2 + r^2\sum_{j=1}^{n}x_{ij}\right)\\ &= \left(-\sum_{i=1}^{n} l_i\right) + r\cdot\left(\sum_{i=1}^{n} a_i\right),\end{aligned}$$

which is a linear function in r and is unbounded. In this case the socially optimal effort does not exist.

4.2 Voluntary Participation in RP

As investment in security is a non-excludable public good, an agent can benefit even if it chooses not to participate in an incentive mechanism. As a result, designing a mechanism which incentivizes the agents to voluntarily participate and exert socially optimal effort levels is not straightforward. In [20] it was shown that no taxation mechanism is able to implement the socially optimal solution while guaranteeing both weak budget balance and voluntary participation. For this reason, it is important to check whether agents will voluntarily participate in resource pooling. In what follows, we first define this notion and then show that under resource pooling the voluntary participation property is satisfied.

*Definition 4.2 (**Voluntary Participation (VP)**).* Consider game G_{rp}^k where agent k opts out of RP and only invests in himself and nobody else invest in agent k ($e_{kj} = e_{jk} = 0,\ \forall j \neq k$), while other agents participate in RP. Let $\mathring{E} = [\mathring{e}_{ij}]_{n\times n}$ be the NE of game G_{rp}^k and $v_i(\mathring{E})$ be the utility of agent i at the NE. We say that resource pooling has the voluntary participation property with respect to agent k, if

$$v_k(\mathring{E}) \leq v_k(\hat{E}), \tag{17}$$

where $\hat{E}$ is the effort profile at the NE of game G_{rp}.[3] If the above is true for all $k \in \mathcal{V}$, then we say that resource pooling has the voluntary participation property.

By the definition of NE, effort profile $\hat{E} = [\hat{e}_{ij}]_{n\times n}$ satisfies the following,

$$v_i(\hat{\boldsymbol{e}}_i, \hat{\boldsymbol{e}}_{-i}) \geq v_i(\boldsymbol{e}_i, \hat{\boldsymbol{e}}_{-i})\ \forall \boldsymbol{e}_i \in R^n, \tag{18}$$

where $\hat{\boldsymbol{e}}_i = [\hat{e}_{i1}, \cdots, \hat{e}_{in}]$. It is worth noting that the above definition does not imply VP defined in (17). The following theorem suggests that resource pooling always satisfies the VP property.

THEOREM 4.3. *If Assumption 2 holds, then agent i achieves higher utility at the NE of game G_{rp}, than his utility at the NE of game G_{rp}^i for all $i \in \mathcal{V}$. That is, resource pooling always satisfies the VP property.*

Proof. See Appendix. ■

As no one has incentive to deviate from resource pooling unilaterally, resource pooling is a better way to improve social welfare as compared to taxation mechanisms which are not able to satisfy the voluntary participation and budget balance constraint simultaneously [20].

4.3 On the Socially Optimal Outcome of Game G_{rp}

While the NE of the RP-IDS game G_{rp} achieves socially optimal levels of effort defined for the IDS game G, the introduction of resource pooling means that each agent now has a bigger action space, thereby giving rise to a different social optimum for this new game. We next show how this new optimum can be computed. Let $E^* = [e_{ij}^*]_{n\times n}$ be the socially optimal effort profile for the RP-IDS game:

$$\begin{aligned}E^* &= \arg\max_{E\in R_+^{n\times n}} \sum_{i=1}^{n} v_i(E)\\ &= \arg\max_{E\in R_+^{n\times n}} \sum_{i=1}^{n}\left[-l_i + a_iE_i - b_i\cdot(\sum_{j=1}^{n}e_{ji}^2) + E_i\sum_{j=1}^{n}x_{ij}E_j\right].\end{aligned}$$

[3] Under Assumption 2, both G_{rp} and G_{rp}^k have an NE.

The assumption below ensures the existence of a solution.

Assumption 3. $2b_i > n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}),\ \forall i \in \mathcal{V}$

Under Assumption 3, it is easy to check that $g(E) = \sum_{i=1}^{n} v_i(E)$ is strictly concave in E. By the first order condition, E^* satisfies the following:

$$\frac{\partial g(E)}{\partial e_{ii}}|_{E=E^*} = a_i - 2b_i e^*_{ii} + \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0$$

$$\frac{\partial g(E)}{\partial e_{ki}}|_{E=E^*} = a_i - 2b_i e^*_{ki} + \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0$$

$$\implies n \cdot a_i - 2b_i E^*_i + n \cdot \sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j = 0,\ \forall i \in \mathcal{V},$$

$$\implies (2B - n \cdot (X + X^T)) \cdot \begin{bmatrix} E^*_1 \\ \vdots \\ E^*_n \end{bmatrix} = n \cdot \boldsymbol{a}\ . \tag{19}$$

Similar as before, we can show that under Assumption 3, $(2B - n \cdot (X + X^T))$ is invertible. Thus the optimal outcome E^* is given by:

$$\begin{bmatrix} E^*_1 \\ \vdots \\ E^*_n \end{bmatrix} = n \cdot (2B - n \cdot (X + X^T))^{-1} \cdot \boldsymbol{a}$$

$$e^*_{ki} = \frac{a_i}{2b_i} + \frac{\sum_{j=1}^{n}(x_{ij} + x_{ji}) \cdot E^*_j}{2b_i},\ \forall i, k \in \mathcal{V} \tag{20}$$

By Corollary 2.2, $(2B - n \cdot (X + X^T))^{-1} \cdot \boldsymbol{a} \geq (2B - (X + X^T))^{-1} \cdot \boldsymbol{a}$ which implies that $E^*_i \geq \hat{E}_i,\ \forall i$, i.e., the total effort exerted on behalf of agent i improves under the social optimum compared to that under the NE of game G_{rp}. As before, not all agents may attain higher individual utility under E^* as compared to their utility under NE $\hat{E}$. Examples are provided in the Appendix.

5 CONCLUSIONS

We considered an IDS game with positive externality, and introduced a resource pooling augmented IDS game, the RP-IDS game, to examine the effect of using resource pooling as a mechanism to incentivize higher effort levels by interdependent agents. We showed that (1) resource pooling increases the total effort exerted on behalf of each agent as compere to no resource pooling, (2) each agent experiences higher utility under resource pooling as compared to no resource pooling, (3) social welfare at the NE of the RP-IDS game is higher than the optimal social welfare under the IDS game, and (4) agents voluntarily participate in resource pooling. An interesting future direction is to consider the case where agents only pool their resources within an alliance or coalition.

Appendix: Proofs are given in [1].

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